The Number Line: An Overview

One of the most overlooked tools of the elementary and middle school classroom is the number line. Typically displayed above the chalkboard right above the alphabet, the number line is often visible to children, though rarely used as effectively as it might be. When utilized in the elementary classroom, the number line has often used to help young children memorize and practice counting with ordinal numbers. Less often, perhaps, the number line is used like a ruler to illustrate the benchmark fractions like ½ or ¼. Beyond an illustration for these foundational representations of whole numbers and some fractions, however, the number line is underutilized as a mathematical model that could be instrumental in fostering number sense and operational proficiency among students.

Recently, however, there has been a growing body of research to suggest the importance of the number line as a tool for helping children develop greater flexibility in mental arithmetic as they actively construct mathematical meaning, number sense, and understandings of number relationships. Much of this emphasis has come as a result of rather alarming performance of young learners on arithmetic problems common to the upper elementary grades. For example, a study about a decade ago of elementary children in the Netherlands – a country with a rich mathematics education tradition – revealed that only about half of all students tested were able to solve the problem 64-28 correctly, and even fewer students were able to demonstrate flexibility in using arithmetic strategies. These results, and other research like them, prompted mathematics educators to question existing, traditional models used to promote basic number sense and computational fluency.

Surprising to some, these research findings suggested that perhaps the manipulatives and mathematical models typically used for teaching arithmetic relationships and operations may not be as helpful as once thought. Base-10 blocks, for example, were found to provide excellent conceptual understanding, but weak procedural representation of number operations. The hundreds chart was viewed as an improvement on arithmetic blocks, but it too was limited in that it was an overly complicated model for many struggling students to use effectively. On the other hand, the number line is an easy model to understand and has great advantages in helping students understand the relative magnitude and position of numbers, as well as to visualize operations. As a result, Dutch mathematicians in the 90’s were among the first in the world to return to the “empty number line,” giving this time-tested model a new identity as perhaps the most important construct within the realm of number and operation. Since that time, mathematics educators across the world have similarly turned to this excellent model with great results.

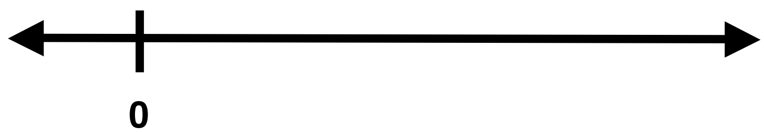
The intent of this book is to share some of the teaching strategies that have emerged in recent years that take advantage of the number line in productive and powerful ways.

The Big Ideas

As noted above, the number line stands in contrast to other manipulatives and mathematical models used within the number realm. Some of the reasons for developing the number line as a foundational tool are illustrated below as key ideas for this textbook.

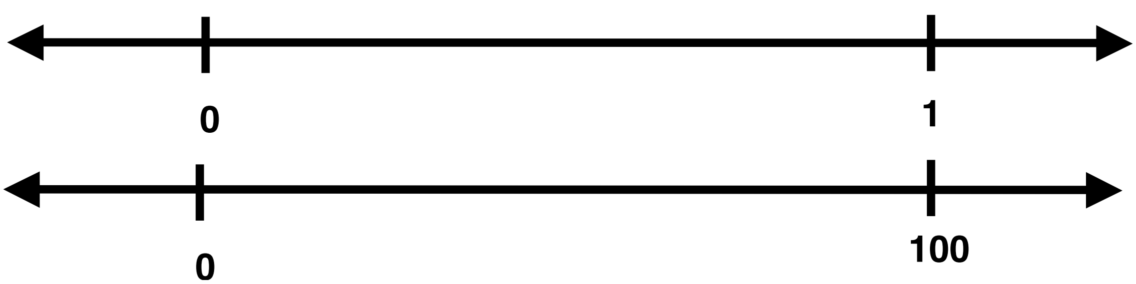
<u>Key Concept #1: The linear character of the number line</u>. The number line is well suited to support informal thinking strategies of students because of its inherent linearity. In contrast to blocks or counters with a “set-representation” orientation, young children naturally recognize marks on a number line as visual representations of the mental images that most people have when they learn to count and develop understanding of number relationships. It is important to note the difference between an “open number line” (shown below) and a ruler with its predetermined markings and scale.

An open number line:



The open line allows students to partition, or subdivide, the space as they see fit, and as they may need, given the problem context at hand. In other words, the number line above could be a starting point for any variety of number representations, two of which are shown below: the distance from zero to 1, or the distance from zero to 100. Once a second point on a number line has been identified, the number line moves from being an open number line, to a closed number line. In addition, the open number line allows for flexibility in extending counting strategies from counting by ones, for example, to counting by tens or hundreds all on the same sized open number lines.

Closed number lines:

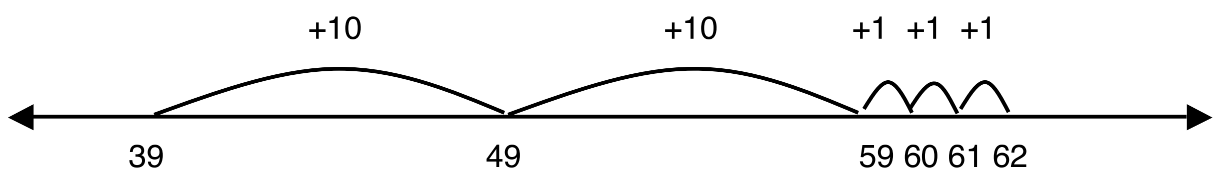


<u>Key Concept #2: Promoting creative solution strategies and intuitive reasoning</u>. A prevalent view in math education reforms is that students should be given freedom to develop their own solution strategies. But to be clear, this perspective does not mean that it is simply a matter of allowing students to solve a problem however they choose. Rather, the models being promoted by the teacher should themselves refine and push the student toward more elegant, sophisticated, and reliable strategies and procedures. This process of formalizing mathematics by having students recognize, discuss, and

internalize their thinking is a key principle in math education reforms, and is one that can be viewed clearly through the use of a model like the number line – a tool that can be used both to model mathematical contexts, but also to *represent methods, thinking progressions, and solution strategies* as well. As opposed to blocks or number tables that are typically cut off or grouped at ten, the open number line suggests continuity and linearity -- a representation of the number system that is ongoing, natural, and intuitive to students. Because of this transparency and intuitive match with existing cognitive structures, the number line is well suited to model subtraction problems, for example, that otherwise would require regrouping strategies common to block and algorithmic procedures.

Key Concept #3: Cognitive engagement. Finally, research studies have shown that students using the empty number line tend to be more cognitively active than when they are using other models, such as blocks, which tend to rely on visualization of stationary groups of objects. The number line, in contrast, allows students to engage more consistently in the problem as they jump along the number line in ways that resonate with their intuitions. While they are jumping on the number line, they are able to better keep track of the steps they are taking, leading to a decrease in the memory load otherwise necessary to solve the problem. For example, imagine a student who is trying to solve the problem: 39 + 23. Under the traditional addition algorithm, or with base-10 manipulatives as well, the "regrouping" strategy, or the "carry the one" algorithm is significantly different cognitively than thinking of this addition problem as a series of jumps. Specifically, the student might represent the problem as: 1) Start at 39; 2) Jump 10; 3) Jump 10 more; 4) jump three. These steps can easily be mirrored on the number line as the student simply traces her thinking with a pencil.

39 + 23 = 62

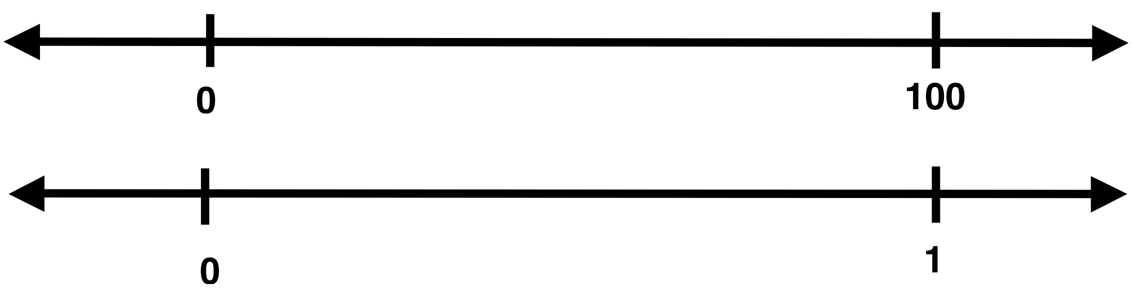


Like any mathematical tool, the more teachers are aware of both the benefits and constraints of the model, the more likely they are to use it effectively with students. Throughout this book, the previous big ideas – though theoretical in nature – are drawn upon repeatedly as students view and subsequently manipulate various open number lines that are used to represent numerous mathematical contexts and operations.

Teaching Ideas

A large portion of this book is devoted to helping students develop a rich sense of numbers and their relationships to one another. The number line is centrally related to this task. As noted above, perhaps the most important teaching point to convey regarding the number line is the notion that, unlike a ruler, it is open and flexible. Given this starting point, students will quickly recognize that they need to create their own actions on the number lines to give the model meaning. Throughout the book, activities include opportunities for students to partition a number line as they see fit. The important thing for students to recognize is that one point alone on a number line does not tell us much about the **scale or magnitude** of numbers being considered. In the number line below, we know very little about this mathematical context other than the fact that it identifies the number 0 on a line.

Yet, by putting a second mark on the line, suddenly each number line below takes on its own significant meaning, and to work with each of these respective lines would require a different kind of mathematical thinking.



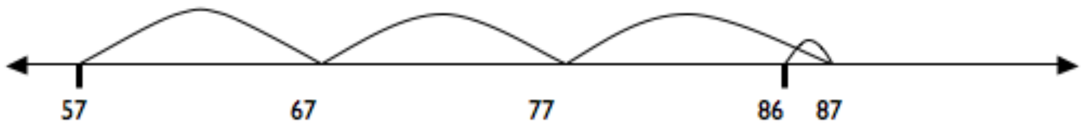
In the first number line above, students will likely begin thinking immediately in terms of tens and twenties — perhaps 50 — as they imagine how they might partition a line from 0 to 100. They will use doubling and halving strategies, among others, as they mark the number line. In the second line, fractional distances between zero and one are likely to come to mind. Once again, students may be using halving strategies if they are finding a number like 1/2 or 1/4. Finding thirds or fifths requires a different type of thinking, of course, which may be beyond K-3 learners. The point here is to recognize the notably different outcomes that might be pursued with these two, simple number lines that each shared a common beginning above (i.e., an open number line with zero identified). Throughout the book, activities will take advantage of this principle.

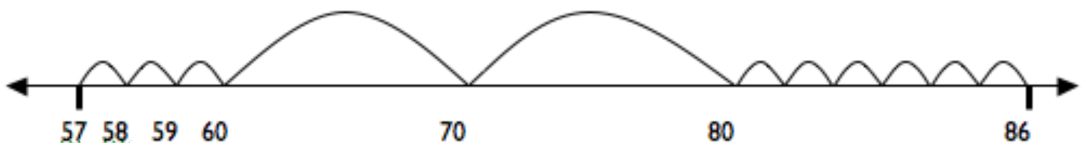
In subsequent sections of the book, the number line is developed as a reliable tool to help students add and subtract. Developed in the book is the idea of a "skip jump" – progression along a number line that is done in specific increments. In this way, the number line becomes a helpful model to mirror how students add and subtract mentally. Students become quite adept at skip jumping by 10's or 100's, for example, and eventually begin to make mental adjustments to the number sentences at hand in order to take advantage of more sophisticated (or for them, easier) intervals for skip jumping. Consider the following problem, for example:

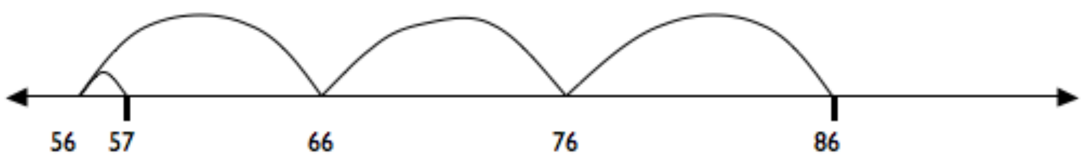
> <u>The Problem</u>: Kerri was trying to set her record for juggling a soccer ball. On her first attempt, she juggled the ball a total of 57 times before it hit the ground. On her second

attempt, she only got a total of 29 juggles. Combining both her first and second attempts, how many times did she juggle the ball in total?

Using a number line flexibly, students may choose to solve this problem in any number of ways, each of which anchors on fundamental understandings of number. The first solution, for example, shows a student who counts on from 57, first by 10's. After skip-jumping forward by 30 (three jumps of 10), the student realizes that she needs to compensate by hopping back by one to arrive at the correct answer → 57 + ***29***.







What are the thinking strategies of the other two students? Take a moment to consider how the number line can be used to mirror the actual thoughts contributing to the solution strategies of these two children… In the second, add 3 to get to an even "decade" number of 60. Now it becomes rather trivial to add 26 to 60. In the third example… take one away (57-1=56), and now we add 30 (instead of 29) to 56 to arrive at a total of 86.

Summary

I have often been asked about the reason for focusing on the number line itself in a book like this rather than on, for example, number concepts, addition, or subtraction. The number line models the natural ways in which we think about all number relationships and number operations. The premise underlying this book-- and the series as a whole-- is that a curriculum best serves students when it provides powerful mathematical tools and understandings that can be used in numerous mathematical contexts and with different types of numbers. Specifically, the tools that students learn to employ in this book can be used with larger whole numbers, integers, fractions, and decimals as well. In short, the number line is an extremely powerful model. The intent is that, perhaps without fully recognizing, students will gain rich intuitions about numbers and operations in this book that will serve them well in years to come.